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In the present note we will discuss some mistakes in [1].

We begin observing that an h^{-2} factor is missing in equation (32), since in this equation we state that we can control

$$\sum_{K\in\mathcal{T}_h} \hat{C}\varepsilon \left\|f\right\|_{L^2(K)},$$

using $\left\|f\right\|_{L^2(\Omega)}$. Clealry, this can not be done because the norm of f inside the sum is not squared. In reality, we meant to control each term of the sum by $\left\|f\right\|_{L^2(\Omega)}$ and observe that we are summing over approximatly $O(h^{-2})$ terms, therefore equation (32) reduces to

$$\left\|u-\hat{u}_h\right\|_h \leq \inf_{\hat{v}_h\in\hat{V}_{h,\varepsilon}} \left\|u-\hat{v}_h\right\|_h + \sum_{K\in\mathcal{T}_h} \hat{C}\varepsilon \left\|f\right\|_{L^2(K)} \leq \inf_{\hat{v}_h\in\hat{V}_{h,\varepsilon}} \left\|u-\hat{v}_h\right\|_h \tag{1}$$

$$+\hat{C}h^{-2}\varepsilon \left\|f\right\|_{L^2(\Omega)}. \tag{2}$$

Since the term $\hat{C}h^{-2}\varepsilon \left\|f\right\|_{L^2(\Omega)}$ appears already in (36) as a consequence of (35), the overall result is not affected by this correction.

In equation (35) we miss-typed the standard VEM interpolation error estimate, using max instead of min. The correct formula is

$$\left\|u-\hat{\Pi}_k u\right\|_h \leq \left\|u-\Pi_k u\right\|_h + \left\|\Pi_k u-\hat{\Pi}_k u\right\|_h \tag{3}$$

$$\leq C(\Omega)h^{\min\{k,m-1\}}\left|u\right|_{H^m(\Omega)} + \left\|f\right\|_{L^2(\Omega)}\hat{C}h^{-2}\varepsilon. \tag{4}$$

This correction leads to correcting also equation (36) as follows

$$\left\|u-\hat{u}_h\right\|_h \leq C(\Omega)h^{\min\{k,m-1\}}\left|u\right|_{H^m(\Omega)} + \left\|f\right\|_{L^2(\Omega)}\hat{C}h^{-2}\varepsilon. \tag{5}$$

References

[1] Trezzi, M. and Zerbinati, U., 2024. When rational functions meet virtual elements: the lightning virtual element method. Calcolo, 61(3), p.35.